# Letters to the Editor

**Discussion of "Results of a Study to Determine the Probability of Chance Match Occurrences Between Fibers Known to be from Different Sources"**

Sir:

In the January 1986 issue of this journal, Fong and Inami reported on a very interesting and worthwhile study [1]. Unfortunately, the section of their paper entitled "Computation of Probabilities" contains many errors which this letter will attempt to point out and correct.

The errors begin with Fong and Inami's Table 3 which is reprinted here for ease of reference. Although Fong and Inami make no mention of how they calculated the figures presented for the probability of chance match occurrences, we suspect that they merely divided the number of match occurrences found by the number of fibers in the study. This is *not* the correct way to do this calculation.

For the proper calculation, three numbers are required: the number of match occurrences, the number of fiber *comparisons* made, and the average number of distinguishable fibers from each source. We are somewhat hampered in our attempt to correct Table 3 by certain problems with Fong and Inami's data. First, the number of match occurrences they found is unclear. Their Table 1 shows a total of 20 matches, 9 of which involved fibers other than blue cotton. In Table 3, the numbers 17 and 6 are used and it is stated in a footnote that 2 matches involving colorless delustered polyester control fibers were not included. No reasons for this are given and such an exclusion does not, at first glance, appear to us to be justified. The remaining discrepancy between Tables 1 and 3 is still unexplained. Thus, it is unclear which of the following three sets of numbers of matches should appear in a revised Table 3: (0,6,17), (0,8,19), or (0,9,20). For purposes of the subsequent calculations, we will use the middle set, (0,8,19).

The second problem concerns the effective number of fiber comparisons performed corrected for source, which Fong and Inami state in a footnote to their Table 3 is 283 882. Unfortunately, they do not show how they arrived at this number. Our calculations[1] indicate that for Class 3 there would have been less than 283 806 comparisons. However, since the numbers are reasonably close, we will use their number in the calculations that follow. Let us now consider the effective number of fiber comparisons performed with the "Class 2 fibers" when the 59 blue cotton fibers are excluded. This would leave a total of $763 - 59 = 704$

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[1]Suppose 3 garments had 20 fibers and 37 garments had 19 fibers for a total of 763 fibers. Now the number of comparisons is

$$\frac{(763)(762)}{2} - (3)\left[\frac{(20)(19)}{2}\right] - (37)\left[\frac{(19)(18)}{2}\right] = 283\ 806.$$

We do not think it is possible to obtain more than 283 806 comparisons. For example, if 1 garment has 21 fibers, 2 have 20 fibers, 36 have 19 fibers, and 1 has 18 fibers, we would have

$$\frac{(763)(762)}{2} - \frac{(21)(20)}{2} - (2)\left[\frac{(20)(19)}{2}\right] - (36)\left[\frac{(19)(18)}{2}\right] - \frac{(18)(17)}{2} = 283\ 804.$$

Therefore, there must have been less than 283 806 comparisons, because the reported standard deviation was 6.4, whereas the case of 3 garments with 20 fibers and 37 garments with 19 fibers yields a standard deviation of only 0.27!

TABLE 3—*The number of match occurrences and the probability of chance match occurrences determined from the intercomparison of 763 fibers from 40 different sources given by probability class.*[a]

| Probability Class | Match Occurrences Found | Calculated Probability by Source |
|---|---|---|
| 1. Fiber type Incidence less than 10%[b] | 0 | 1/1000 |
| 2. All fibers less blue cotton | 6[c] | 1/130 |
| 3. All fibers | 17[c] | 1/46 |

[a]Average number of fibers from each source = 19.1, standard deviation = 6.4. Effective number of fiber comparisons performed corrected for source = 283 882.
[b]See Table 1 for fiber types having incidence less than 10%.
[c]Two matches involving colorless, delustered polyester control fibers not included.

fibers for an average of 17.6 fibers per garment. If these are evenly distributed, we would have 16 garments with 17 fibers each and 24 garments with 18 fibers each to yield

$$\frac{(704)(703)}{2} - (16)\left[\frac{(17)(16)}{2}\right] - (24)\left[\frac{(18)(17)}{2}\right] = 241\ 608 \text{ comparisons.}$$

With the "Class 1" fibers, since 0 matches were found, the number of comparisons is immaterial, as is the average number of distinguishable fibers from each source. In the calculations for the "Class 2" and "Class 3" fibers, we will use the whole numbers 18 and 20, respectively, as the average number of fibers from each source.

Once we have the required three numbers, the calculations for Table 3 can be done in three ways. Since it does not require any assumption of independence, the preferred method involves use of the Bonferroni Inequality [2] to obtain a conservative upper bound on the probabilities. (For an example of the use of the Bonferroni Inequality in connection with a similar problem, see Ref 3.) We then get the following results:

| Probability Class | Probability |
|---|---|
| 1. Fiber type incidence less than 10% | 0 |
| 2. All fibers less blue cotton | $(18)\left[\dfrac{8}{241\ 608}\right] = \dfrac{1}{1678}$ |
| 3. All fibers | $(20)\left[\dfrac{19}{283\ 882}\right] = \dfrac{1}{747}$ |

Similar results can be obtained by the other two methods.
Assuming independence, the binomial probabilities would be:

$$\text{Class 2} \quad 1 - \left[\frac{241\ 600}{241\ 608}\right]^{18} = \frac{1}{1678} \quad \text{and}$$

$$\text{Class 3} \quad 1 - \left[\frac{283\ 863}{283\ 882}\right]^{20} = \frac{1}{748},$$

and the Poisson approximations to the binomial would be:

$$\text{Class 2} \quad 1 - \exp[-(18)(8)/(241\ 608)] = \frac{1}{1678} \quad \text{and}$$

$$\text{Class 3} \quad 1 - \exp[-(20)(19)/(283\ 882)] = \frac{1}{748}.$$

The five examples discussed by Fong and Inami are also in error for three reasons. First there are the previously discussed errors in their Table 3. Second, some of the calculations used in the examples are in error. Third, the examples refer to what Fong and Inami termed "class 1 and class 2 fibers." Since we are dealing with average probabilities, it is not legitimate to exclude those fibers most likely to match. Accordingly, the only proper probability to use is what Fong and Inami term "Class 3 fibers," that is, all fibers. In the following reworking of the examples we will therefore consider only "Class 3 fibers." Since the denominator used in the probability calculation for our revised Table 3 may be too large, we will round the probability of a chance match occurrence down to 1/700.

Example 1—One match between Source 1 and Source 2, each of which has 20 distinguishable fibers by type and color.

Probability of at least one match, using Bonferroni's Inequality is 20 × 1/700 = 1/35.

Example 2—Same as Example 1 except that Source 2 involves only a single distinguishable fiber.

This is the probability we calculated for Table 3, that is, 1/700.

Example 3—Same as Example 1 except that both sources yield one distinguishable fiber. This is 20/283 882 ≃ 1/14 000.

Example 4—Same as Example 1 except that 3 matches are found: If Source 2 fibers are independent, we would have

$$\frac{(20)(19)(18)}{(3)(2)(1)} \left[\frac{1}{700}\right]^3 \left[\frac{699}{700}\right]^{17} = \frac{1}{308\ 279}$$

for exactly 3 matches. But wouldn't it be better to ask for the probability of at least 3 matching fibers? This would give

$$1 - \left[\frac{699}{700}\right]^{20} - (20)\left[\frac{1}{700}\right]\left[\frac{699}{700}\right]^{19} - \frac{(20)(19)}{2}\left[\frac{1}{700}\right]^2\left[\frac{699}{700}\right]^{18} = \frac{1}{306\ 407}.$$

Example 5—Two matches between Source 1 (20 distinguishable fibers) and Source 2 (40 distinguishable fibers).

Let us concentrate on the recommended approach. What is the probability of at least 2 matches, assuming only that Source 2 fibers are independent? This would be

$$1 - \left[\frac{699}{700}\right]^{40} - (40)\left[\frac{1}{700}\right]\left[\frac{699}{700}\right]^{39} = \frac{1}{651}.$$

Fong and Inami contend in the discussion section of their paper that: "if dependency is admitted then the proof that the two articles were in contact is also effectively admitted." This is fallacious. If green polyester and blue polyester tend to occur frequently together (that is, more frequently than expected under the independence assumption), then finding

both green polyester and blue polyester fibers that match would not increase the weight of evidence as much as would be implied by the independence assumption. However, if green polyester and purple nylon occur together less frequently than expected under an independence assumption and they both match the suspect garment, the weight of evidence is stronger that the garment in question was associated with the scene of the crime. However, as we have seen, this lack of independence is not a problem when the probability calculations are based on the Bonferroni Inequality.

In the acknowledgment section of their paper, Fong and Inami refer to the risk of courting criticism in the publication of studies such as theirs. We wish to emphasize that criticism is not the intention of this letter. Rather we write in an attempt to improve Fong and Inami's paper and to make a general contribution to the discussion of statistical evaluation of forensic science evidence.

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*References*

[1] Fong, W. and Inami, S. H., "Results of a Study to Determine the Probability of Chance Match Occurrences Between Fibers Known to be from Different Sources," *Journal of Forensic Sciences*, Vol. 31, No. 1, Jan. 1986, pp. 65–72.
[2] Feller, W., *An Introduction to Probability Theory and Its Applications Volume I*, 3rd ed., John Wiley and Sons, New York, 1968, p. 110.
[3] Gaudette, B. D., "A Supplementary Discussion of Probabilities and Human Hair Comparisons," *Journal of Forensic Sciences*, Vol. 27, No. 2, April 1982, pp. 279–289.

**Author's Response**

Sir:

Mr. Gaudette's letter to the editor is appreciated. This is especially so since by providing its answer I am given an opportunity to engage in an open dialogue on identified issues. This can be helpful to all who are concerned with decision making in the practice of forensic criminalistics.

*Independence*

I am bemused by Mr. Gaudette's polemic usage of the term "fallacious" and unpersuaded by his argumentation with the hypothetical example of green polyester and blue polyester fibers.

*If the hypothesis tendered is in fact real then its reality should be uncovered by a study such as we conducted. This is one of the reasons that a frequency of occurrence study is performed.*

*Calculations*

A reexamination of Table 1 of our paper [1] shows that the total number of matches was 20. Two of these involved colorless delustered polyester control fibers and were omitted to give a total of 18.

*The rationale for the omission was that in an actual case such matches could not be assigned significant meaning. To do so would be mindless especially in consideration that the basis of the comparisons in the study was microscopic. If there is difficulty in grasping the fundamental reasoning involved here one should imagine the foolishness of comparing white cotton fibers recovered from two T-shirts in an actual case when it is known that the control fibers from each consist of white cotton fibers!*

We are grateful for having it pointed out that if blue cotton fibers are to be omitted a correction must be made. Accordingly, for probability Class 2 fibers the average number of fibers per source would be 17.6 and not 19.

Mr. Gaudette expends extraordinary effort towards recalculating the number which we give for the number of effective intercomparisons made. For some inexplicable reason he finds it difficult to accept the number which we report which was 283 882.

*In our paper and on p. 66 in the third paragraph under* Experimental Procedures *we state: A log was maintained to provide a source-to-source accounting of the number of fibers characterized. This was done to permit the accurate computation of the real number of intercomparisons required. The footnote at the bottom of our Table 3 states: Effective number of fiber comparisons performed corrected for source = 283 882.*

*I am beguiled in my spare time by thoughts of why Mr. Gaudette would think that by a calculation he can arrive at a more accurate value than ours when ours was a real number arrived at through a log recording by source and subtraction of the comparisons between fibers from the same source on an ongoing source-by-source basis.*

We arrived at our value of $p$ by dividing the number of match occurrences found by the number of comparisons by source. The latter number is given by: $n(n - 1)/2$. Since there was 40 sources the number of comparisons by source was 780. Mr. Gaudette is in error when he says," . . . we suspect that they merely divided the number of match occurrences found by the number of fibers in the study."

*Whether our's or Gaudette's approach towards calculation is applied the answers to be obtained are virtually the same. For example, using our approach I calculate the probabilities of hair matchings based upon data given in the article describing a similar study on hair by Gaudette and Keeping [2] as being 1/4950 for the chance occurrence of a single hair from Source 1 matching 1 of 9 dissimilar hairs from Source 2, and 1/44 551 for the chance occurrence of matching single hairs from Sources 1 and 2. Gaudette and Keeping give for these two situations: 1/4 500 and 1/40 737, respectively. Application of the Bonferroni Inequality changes the value of 1/4 500 only minutely (to 1/4 526) as pointed out by Gaudette [3]. The value for our* p *that I would calculate for their hair study would be 1/550.*

*I favor our method because it is* source oriented *(important to the criminalist), flexible, readily understood, and the final results differ by an insignificant amount as compared with those obtained from the nonsource oriented and needlessly obscure approach used by Gaudette.*

Eleven blue cotton fiber matches were further subtracted to yield a final total of seven matches for computation under our Class 2 category of fibers.

*The rationale for this subtraction was based upon intuitive logic and given in our paper. The presentation through probability classes was such that workers could use their own judgment as to the best approach. Our view is that the assigning of equal weight to all fiber matches implicit in the "average probabilities" approach favored by Gaudette is contrary to good sense. If there is difficulty in comprehending the logic of this assertion one needs only to consider that strict adherence to his dogma would require that the yellow-green nylon*

*fibers match in the Wayne Williams case* [4] *be assigned the same evidentiary weight as a blue cotton fibers match.*

*Also, most of those who do both hair and fiber comparisons in case work would have a difficult time accepting Gaudette's calculation of our Example 5 as having a probability of 1/651 (as opposed to our calculated 1/500 000) since this would be tantamount to accepting that two fiber matches, for example, yellow-green nylon fibers and violet acetate fibers (such as found in the Wayne Williams case) have an evidentiary value considerably less than the finding of a single hair match according to conditions specified in the work Gaudette co-authored with Keeping. In their work, a probability of 1/4 500 was arrived at for the likelihood of one match (1 hair from Source 1 found to match 1 of 9 dissimilar hairs from Source 2). The meaning here is that a hair match, according to Gaudette's calculations (under the conditions given), has an evidentiary value nearly ×7 that of two of the critical kinds of fiber matches in the Wayne Williams case.* Now that is really something!

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*References*

[1] Fong, W. and Inami, S. H., "Results of a Study to Determine the Probability of Chance Match Occurrences Between Fibers Known to be from Different Sources," *Journal of Forensic Sciences,* Vol. 31, No. 1, Jan. 1986, pp. 65–72.
[2] Gaudette, B. D. and Keeping, E. S., "An Attempt at Determining Probabilities in Human Scalp Hair Comparison," *Journal of Forensic Sciences,* Vol. 19, No. 3, July 1974, pp. 599–606.
[3] Gaudette, B. D., "A Supplementary Discussion of Probabilities and Human Hair Comparisons," *Journal of Forensic Sciences.* Vol. 27, No. 2, April 1982, pp. 279–289.
[4] Deadman, H. A., "Fiber Evidence and the Wayne Williams Trial," Part I, *FBI Law Enforcement Bulletin,* Vol. 53, No. 3, March 1984, pp. 12–20 and Part II, *FBI Law Enforcement Bulletin,* Vol. 53, No. 5, May 1984, pp. 10–19.